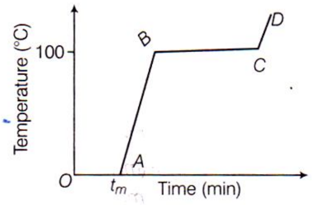
WEEKLY TEST TYJ -1 TEST - 30 R
SOLUTION Date 08-12-2019

[PHYSICS]

1. ***(d)*** A plot of temperature *versus* time showing the changes in the state of ice on heating (not to scale). (Also *refer solution no.*117.



$O \to A$: solid + liquid

$A \to B$: liquid

$B \to C$: liquid + gas

$C \to D$: gas

2. ***(a)*** The change of state from solid to liquid is called **melting** and from liquid to solid is called **fusion.** It is observed that the temperature remains constant until the entire amount of the solid substance melts. *e.g.*, Both the solid and liquid states of the substance coexist in thermal equilibrium during the change of states from solid to liquid. The temperature at which the solid and the liquid states of substance are in thermal equilibrium with each other is called its **melting point.**

3. -

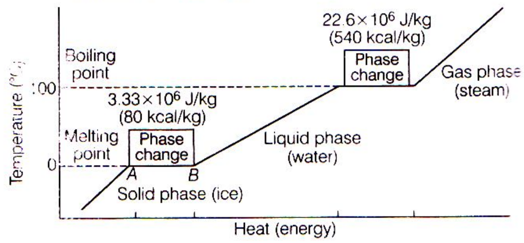
4 ***(a)*** The heat required during a change of state depends upon the heat of transformation and the mass of the substance undergoing a change of state. Thus, if mass *m* of a substance undergoes a change from one state to the other, then the quantity of heat required is given by

$$Q = mL \quad \text{or} \quad L = Q/m$$

where, *L* is known as latent heat and is a characteristic of the substance. Its SI unit is J kg^{-1}.

The value of *L* also depends on the pressure. Its value is usually quoted at standard atmospheric pressure.

5. ***(c)*** The latent heat for a solid-liquid state change is called the **latent heat of fusion** (L_f), and that for a liquid-gas state change is called the **latent heat of vaporisation** (L_v). A plot of temperature *versus* heat energy for a quantity of water is shown in figure.



For 1 kg mass $H_B - H_A$ = Latent heat of fusion.

6. ***(a)*** Heat lost by water $= m_i s_w (\theta_i - \theta_f)_w$

$= (0.30\text{ kg})(4186\text{ J kg}^{-1}\text{ K}^{-1})(50.0\ °C - 6.7\ °C)$

$= 54376.14$ J

Heat required to melt ice $= m_i L_f = (0.15\text{ kg})\, L_f$

Heat required to raise temperature of ice water to final temperature $= m_i s_w (\theta_f - \theta_i)_f$

$= (0.15\text{ kg})(4186\text{ J kg}^{-1}\text{ K}^{-1})(6.7\ °C - 0\ °C)$

$= 4206.93$ J

Heat lost = Heat gained.

$54376.14\text{ J} = (0.15\text{ kg})\, L_f + 4206.93\text{ J}$

$L_f = 3.34 \times 10^5\ \text{J-kg}^{-1}$

7. **(b)** We have, mass of the ice $m = 3\,kg$

Specific heat capacity of ice, $Sice = 2100\,J\,kg^{-1}K^{-1}$

Specific heat capacity of water, S_{water}
$= 4186\,J\,kg^{-1}K^{-1}$

Latent heat of fusion of ice, $S_{ice} = 3.35 \times 10^5\,J\,kg^{-1}$

Latent heat of steam, $L_{steam} = 2.256 \times 10^6\,J\,kg^{-1}$

Now, Q = heat required to convert 3 kg of ice at −12°C to steam at 100°C.

Q_1 = Heat required to convert ice at −12°C to ice at 0°C

$= mS_{ice}\,\Delta T_1 = 3 \times 2100 \times [0-(-12)]°C = 75600\,J$

Q_2 = Heat required to melt ice at 0°C to water at 0°C.

$= mL_{ice} = 3 \times (3.35 \times 10^5\,J\,kg^{-1}K^{-1})$

$= 1005000\,J$

Q_3 = Heat required to convert Water at 0°C to water at 100°C.

$= ms_w\Delta T_2 = (3\,kg)(4186\,J\,Kg^{-1}K^{-1}) \times (100°C)$

$Q_3 = 1255800\,J$

Q_4 = Heat required to convert water at 100°C to steam at 100°C

$= mL_{steam} = 3 \times (2.256 \times 10^6\,J\,kg^{-1}K^{-1}) = 6768000\,J$

So, $Q = Q_1 + Q_2 + Q_3 + Q_4$

$= 75600\,J + 1005000\,J + 1255800\,J + 6768000\,J$

$= 9.1 \times 10^6\,J$

8. **(b)** Here, $m = 60\,kg = 60 \times 10^3 g$, $c = 0.83\,cal\text{-}g^{-1}\,°C^{-1}$

$Q = 200\,kcal = 2 \times 10^6\,cal$

Amount of heat required for a person.

$\therefore \quad Q = mc\Delta T$

$$\Rightarrow \quad \Delta T = \frac{Q}{mc} = \frac{2 \times 10^6}{60 \times 10^3 \times 0.83}$$

$= 40.16°C$

9. **(b)** Heat lost by water in cooling from 25°C to 10°C

$Q = mc\Delta T = 200 \times 1 \times (25-10) = 3000\,cal$

Here, gained by ice at −14°C to change into water at 10°C.

$Q = (mc\Delta T)_{ice} + mL + (mc\Delta T)_{water}$

$= m \times 0.5 \times 14 + m \times 80 + m \times 1 \times 10$

$= 97\,m\,cal$

According to principle of calorimetry, $97\,m = 3000$

$$\text{Mass of ice } (m) = \frac{3000}{97} = 31\,g$$

10. **(b)** The rate of loss of heat depends on the difference in temperature between the body and its surroundings. Also, *refer to solution.no.* 183.

11. **(c)** According to Newton's law of cooling, the rate of loss of heat, $-dQ/dt$ of the body is directly proportional to the difference of temperature $\Delta T = (T_2 - T_1)$ of the body and the surroundings. The law holds good only for small difference of temperature. Also, the loss of heat by radiation depends upon the nature of the surface of the body and the area of the exposed surface. We can write

$$-\frac{dQ}{dt} = k(T_2 - T_1)$$

where, k is a positive constant depending upon the area and nature of the surface of the body. Suppose a body of mass m and specific heat capacity s is at temperature T_2. Let T_1 be the temperature of the surroundings. If the temperature falls by a small amount dT_2 in time dt, then the amount of heat lost is

$$dQ = ms\,dT_2$$

∴ Rate of loss of heat is given by

$$\frac{dQ}{dt} = ms\frac{dT_2}{dt}$$

From equation, $-\dfrac{dQ}{dt} = k\,(T_2 - T_1)$

and $\dfrac{dQ}{dt} = ms\dfrac{dT_2}{dt}$

$$\Rightarrow \quad -ms\frac{dT_2}{dt} = k\,(T_2 - T_1)$$

$$\Rightarrow \quad \frac{dT_2}{T_2 - T_1} = \frac{k}{ms}dt = -k\,dt$$

where, $K = k/ms$

On integrating, $\log_e\,(T_2 - T_1) = -Kt + C$

$\Rightarrow \quad T_2 = T_1 + C'e^{-Kt}$, where $C' = e^C$

Above equation enables to calculate the time of cooling of a body through a particular range of temperature.

12. **(c)** The loss of heat by radiation depends upon the nature of surface of the body and the area exposed surface.

Also, *refer to solution no.* 186.

Heat radiated per unit time, by body

$$= \text{Heat current} = H = \frac{\Delta Q}{\Delta t} = \varepsilon\sigma AT^4$$

Here, ε = Emissivity if body depend on nature of suface of body

A = Exposed area of the body

σ = Stefan-Boltzmann constant

T = Temperature of body

If surrounding temperature is T_s, then net loss of thermal energy by body per unit time $= \varepsilon\sigma A\,(T^4 - T_5{}^4)$.

13. ***(d)*** In first case, $T_1 = 60°C$, $T_2 = 40°C$

$T_0 = 10°C, t = 7$ min $= 420$ s.

According to Newton's law of cooling, we get

$$mc\,\frac{T_1 - T_2}{t} = k\left[\frac{T_1 + T_2}{2} - 10\right]$$

$$mc\,\frac{(60-40)}{420} = k\left[\frac{60+40}{2} - 10\right]$$

$$mc \times \frac{20}{420} = k \times 40$$

In second case, $T_1 = 40°C, T_2 = ?, T_0 = 10°C$

and $t = 7$ min $= 420$ s

$$mc \times \frac{40 - T_2}{420} = k\left[\frac{40+T_2}{2} - 10\right]$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{20}{40 - T_2} = \frac{40}{\frac{40+T_2}{2} - 10}$$

$$20 + \frac{T_2}{2} - 10 = 80 - 2T_2$$

On solving, we get $T_2 = 28°C$.

14. ***(c)*** Power radiated *i.e.*, $E = A\sigma T^4 = 4\pi r^2 \sigma T^4$

When radius is halved and temperature is doubled, power radiated becomes,

$$E' = 4\pi\left[\frac{r}{2}\right]^2 \times \sigma\ (2T)^4 = 4 \times 4\pi r^2\ \sigma T^4 = 4E$$

$$= 4 \times 450 = 1800\ \text{W}$$

15. ***(a)*** Here, in Ist case, $T_1 = 81°C$, $T_2 = 79°C$, $T_0 = 30°C$ and $t = 1$ min. As fall in temperature, in accordance with Newton's law of cooling expression is

$$-\frac{dT}{dt} = K(T - T_0), \text{ we can write}$$

$$\left(\frac{T_1 - T_2}{t}\right) = -K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

$$\frac{81-79}{1\,\text{min}} = -K\left[\frac{81+79}{2} - 30\right]$$

$$\Rightarrow \quad \frac{2}{1\,\text{min}} = -K \times 50 \quad \ldots(i)$$

and in 2nd case, $T_1' = 61°C$, $T_2' = 59°C$. If time of cooling be t', then

$$\frac{61-59}{t'} = K\left[\frac{61+59}{2} - 30\right] \text{ or } \frac{2}{t'} = -K \times 30 \quad \ldots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$t' = \frac{50}{30}\text{ min} = \frac{5}{3}\text{ min} = 1\text{ min } 40\text{ s}$$

16. ***(b)*** When a metallic rod is heated it expands. Its moment of inertia (I) about a perpendicular bisector increases. According to law of conservation of angular momentum, its angular speed (ω) decreases, since $\omega \propto 1/I$.

17. ***(b)*** According to linear expansion, we get

$$L = L_0\,(1 + \alpha\Delta\theta)$$

$$\frac{L_1}{L_2} = \frac{1 + \alpha\,(\Delta\theta_1)}{1 + \alpha\,(\Delta\theta_2)} = \frac{10}{L_2}$$

$$= \frac{1 + 11 \times 10^{-6} \times 20}{1 + 11 \times 10^{-6} \times 19}$$

$$\Rightarrow \quad L_2 = 9.99989$$

Length is shorter by

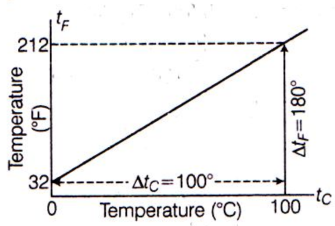
$$= 10 - 9.99989 = 0.00011 = 11 \times 10^{-5}\ \text{cm}$$

18. ***(a)*** Here, coefficient of volumetric expansion *i.e.*,

$$\rho = \frac{\Delta V}{V \times \Delta T} = \frac{0.24}{100 \times 40} = 6 \times 10^{-5}/°C$$

$$\Rightarrow \quad \alpha = \frac{\rho}{3} = 2 \times 10^{-5}/°C$$

19 ***(d)*** A relationship for converting between the two scales may be obtained from a graph of Fahrenheit temperature (t_F) *versus* Celsius temperature (t_C) in a straight line whose equation is



$$\frac{t_F - 32}{180} = \frac{t_C}{100}$$

20. ***(d)*** Let initial temperature in Fahrenheit and Celsius scale be t_{F_1} and t_{C_1}, respectively and the final temperature be t_{F_2} and t_{C_2}, respectively.

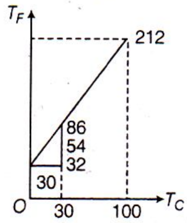
From relation, $\dfrac{t_F - 32}{180} = \dfrac{t_C}{100}$

or, $$\frac{t_{F_1} - 32}{180} = \frac{t_{C_1}}{100} \quad \ldots(i)$$

or, $$\frac{t_{F_2} - 32}{180} = \frac{t_{C_2}}{100} \quad \ldots(ii)$$

Subtracting Eq. (i) from Eq. (ii),

$$\frac{(t_{F_2} - t_{F_1})}{180} = \frac{t_{C_2} - t_{C_1}}{100}$$



Given, $t_{C_2} - t_{C_1} = 30\,°C$

$$\Rightarrow \quad t_{F_2} - t_{F_1} = \frac{180}{100} \times 30°F = 54°F$$

[CHEMISTRY]

21. Higher the oxidation number of central aotm in oxoacid, higher is the acidic character.

22. HIO_4 H_3IO_5 H_5IO_6

$1 + x - 8 = 0$ $3 + x - 10 = 0$ $5 + x - 12 = 0$

$x = +7$ $x = +7$ $x = +7$

23. Oxidation number of Br_2 (element form) = 0

In BrO_3^-, O.N. of Br + 3(–2) = –1

$\Rightarrow$ O.N. of Br = +5

24. $H_4P_2O_5$: $4 + 2x - 10 = 0$

$\Rightarrow x = +3$

$H_4P_2O_6$: $4 + 2x - 12 = 0$

$\Rightarrow x = +4$

HP_2O_7: $4 + 2x - 14 = 0$

$\Rightarrow x = +5$

26. Lower the reduction potential, better the reducing power. Hence, the correct choice is 'd'

27. $\overset{(0)}{3Cl_2} + 6OH^- \rightarrow \overset{(-1)}{5Cl^-} + \overset{(+5)}{ClO_3^-} + 3H_2O$

28. Higher the reduction potential easier to gain electrons.

Lower the reduction potential easier the loss of electrons.

30. Valence is combining capacity. Hence, valence of sulphur is '2'. Oxidation number is apparent charge which is zero for form O.N. of sulphur element is zero.

32. In $C_2O_4^{2-}$, C-atom has O.N. + 3. In CO_2, C-atom has O.N. +4. Hence, $C_2O_4^{2-}$ is reducing agent

37. Higher the reduction potential, easier is the gain of electrons.

[MATHEMATICS]

41. (c) $\lim_{n\to\infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)} = \lim_{n\to\infty} \frac{4n^3+4n^2+n}{n^3+5n^2+5n-2}$

$$= \lim_{n\to\infty} \frac{n^3\left(4+\frac{4}{n}+\frac{1}{n^2}\right)}{n^3\left(1+\frac{5}{n}+\frac{5}{n^2}-\frac{2}{n^3}\right)} = 4$$

42. (b) $\lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}.$

43. (b) $\lim_{x\to a} \frac{\sqrt{3x-a}-\sqrt{x+a}}{x-a}$

$$= \lim_{x\to a} \frac{\sqrt{3x-a}-\sqrt{x+a}}{(x-a)} \times \frac{\sqrt{3x-a}+\sqrt{x+a}}{\sqrt{3x-a}+\sqrt{x+a}}$$

$$= \frac{2}{2\sqrt{2a}} = \frac{1}{\sqrt{2a}}$$

Aliter : Apply L-Hospital's rule

$$\lim_{x\to a} \frac{\sqrt{3x-a}-\sqrt{x+a}}{x-a} = \lim_{x\to a} \frac{3}{2\sqrt{3x-a}} - \frac{1}{2\sqrt{x+a}}$$

$$= \frac{3}{2\sqrt{2a}} - \frac{1}{2\sqrt{2a}} = \frac{1}{\sqrt{2a}}.$$

44. (b) $\lim_{x\to 2} \frac{x^n - 2^n}{x-2} = n.2^{n-1} \Rightarrow n.2^{n-1} = 80 \Rightarrow n = 5.$

45. (a) $\lim_{x\to 0} \frac{x.2\sin^2 x}{x^2} = 2.\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \cdot \lim_{x\to 0} x = 0.$

46. (c) $\left[\lim_{x\to\infty}\left(1+\frac{1}{x/2}\right)^{x/2}\right]^2 = e^2.$

47. (a) $\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)\times(\sqrt{x}+1)}{(x-1)(2x+3)\times(\sqrt{x}+1)} = \frac{-1}{5.2} = \frac{-1}{10}.$

48. (d) $\lim_{x\to 0} \frac{2\sin 2x}{2x} = 2.$

49. (a) $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{2\sqrt{x}}.$

Aliter : Apply L-Hospital rule,

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{1}{2\sqrt{x+h}} = \frac{1}{2\sqrt{x}}.$$

50. (b) $\lim_{x\to 0} \frac{2^x - 1}{(1+x)^{1/2}-1} = \lim_{x\to 0} \frac{2^x \log 2}{\frac{1}{2}(1+x)^{-1/2}}$

$$\left\{\because \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}\right\}$$

$$= 2\log 2 = \log 4.$$

51. (c) $\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx} = \lim_{x\to 0} \left\{\frac{2\sin^2 \frac{mx}{2}}{2\sin^2 \frac{nx}{2}}\right\}$

$$= \lim_{x\to 0} \left[\left\{\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right\}^2 \frac{m^2x^2}{4} . \frac{1}{\left\{\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right\}^2} . \frac{4}{n^2x^2}\right]$$

$$= \frac{m^2}{n^2} \times 1 = \frac{m^2}{n^2}.$$

Aliter : Apply L-Hospital's rule,

$$\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx} = \lim_{x\to 0} \frac{m\sin mx}{n\sin nx} = \lim_{x\to 0} \frac{m^2\cos mx}{n^2\cos nx} = \frac{m^2}{n^2}.$$

52. (c) $\lim_{x\to 1} \frac{x-1}{(x-1)(2x-5)} = -\frac{1}{3}.$

Aliter : Apply L-Hospital's rule.

53. (b) Apply L-Hospital's rule, $\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

$$= \lim_{x\to 0} \frac{\cos x}{2\sqrt{1+\sin x}} + \frac{\cos x}{2\sqrt{1-\sin x}} = \frac{1}{2} + \frac{1}{2} = 1.$$

54. (a) $\lim_{\alpha\to\pi/4} \frac{\sin\alpha - \cos\alpha}{\alpha - \pi/4}$

$$= \lim_{\alpha\to\pi/4} \left\{\frac{\sqrt{2}\left(\sin\alpha.\frac{1}{\sqrt{2}} - \cos\alpha.\frac{1}{\sqrt{2}}\right)}{\left(\alpha - \frac{\pi}{4}\right)}\right\}$$

$$= \sqrt{2} \lim_{\alpha \to \pi/4} \frac{\sin\left(\alpha - \frac{\pi}{4}\right)}{\left(\alpha - \frac{\pi}{4}\right)} = \sqrt{2} \times 1 = \sqrt{2}.$$

Aliter : Apply L-Hospital's rule,

$$\lim_{\alpha \to \pi/4} \frac{\sin\alpha - \cos\alpha}{\alpha - (\pi/4)} = \lim_{\alpha \to \pi/4} \frac{\cos\alpha + \sin\alpha}{1} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}.$$

55. (c) $\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x} = \lim_{x \to 0} \left\{ \frac{\frac{2\tan 2x}{2x} - 1}{3 - \frac{\sin x}{x}} \right\} = \frac{1}{2}.$

Aliter : Apply L-Hospital's rule

$$\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x} = \lim_{x \to 0} \frac{2\sec^2 2x - 1}{3 - \cos x} = \frac{2-1}{3-1} = \frac{1}{2}.$$

56. (a) $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{a}{b} \frac{\sin ax}{ax} \frac{bx}{\sin bx} = \frac{a}{b}.$

57. (b) $\lim_{x \to 0} \frac{\sin x^o}{x} = \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{x} = \frac{\pi}{180}$

$\left\{ \because x^\circ = \frac{\pi x}{180} radian \right\}.$

58. (a) Apply the L-Hospital's rule, $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$

59. (b) $\lim_{x \to 0} \frac{\cos ax - \cos bx}{x^2}$

$$= \lim_{x \to 0} \frac{2\sin\left(\frac{a+b}{2}\right)x . \sin\left(\frac{b-a}{2}\right)x}{\left(\frac{a+b}{2}\right)x . \frac{2}{a+b} . \frac{2}{b-a} . \left(\frac{b-a}{2}\right)x} = \frac{b^2 - a^2}{2}$$

Aliter : Apply L-Hospital's rule,

$$\lim_{x \to 0} \frac{\cos ax - \cos bx}{x^2} = \lim_{x \to 0} \frac{-a\sin ax + b\sin bx}{2x}$$

$$= \lim_{x \to 0} \frac{-a^2\cos ax + b^2\cos bx}{2} = \frac{b^2 - a^2}{2}.$$

60. (a) $\lim_{x \to a} \frac{x^9 + a^9}{x + a} = 9 \Rightarrow \frac{2a^9}{2a} = 9 \Rightarrow a^8 = 9$

$\Rightarrow a = 9^{1/8}$